

Finite width and local field corrections to spin Coulomb drag in a quasi-two-dimensional electron gas

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We study the spin Coulomb drag in a quasi-two-dimensional electron gas of finite transverse width, including local field corrections beyond the random phase approximation (RPA). We find that the finite transverse width of the electron gas causes a significant reduction of the spin Coulomb drag. This reduction, however, is largely compensated by the enhancement coming from the inclusion of many-body local field effects beyond the RPA, thereby restoring good agreement with the experimental observations by C. P. Weber *et al.*, *Nature*, **437**, 1330 (2005).

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Lately a number of spin-based devices have been designed and studied with the goal of combining memory and logical functions in a single device [1, 2, 3, 4, 5]. A crucial requirement for most spin devices is the existence of a robust population of spin-polarized carriers [6] as well as the feasibility of moving the spins in the device by means of spin currents. In this context, understanding the various scattering processes which control the relaxation of the spin and the spin current has emerged as an important theoretical and practical problem. In this Letter we focus on the spin current relaxation caused by electron-electron scattering in quasi-two-dimensional quantum wells of finite width.

It is now well established [5, 8, 9] that the Coulomb interaction induces momentum transfer between the carrier populations of “up” and “down” spin. This results in a spin Coulomb drag (SCD) effect within a single two-dimensional electron gas (2DEG) layer, in close analogy to the conventional charge Coulomb drag, which occurs between spatially separated layers. Because the inherent friction between the two spin components leads to a decay of the spin current even in the absence of impurities, the SCD has recently become a subject of experimental and theoretical investigations [7, 10, 11, 12, 13]. In particular, the SCD is known to reduce the spin diffusion constant relative to the conventional density diffusion constant, and thus to prolong the time during which a spin packet can be effectively manipulated. Indeed a significant reduction in the spin diffusion constant was measured by Weber *et al.* [7] and found to be in quantitative agreement with the SCD reduction calculated for a strictly 2DEG in the random phase approximation (RPA) [10].

The agreement between the RPA theory for an electron gas of zero width and the experiment is encouraging, but also somewhat puzzling. Quantum wells in which the experiments have been performed are not strictly two-dimensional but have finite transverse widths of the or-

der of 10 nm. In contrast to the conventional charge Coulomb drag, where the inter-layer spacing causes an exponential suppression of large angle scattering events, the main contribution to the SCD comes from events with a momentum transfer of the order of the Fermi momentum. At such values of the momentum transfer, the form factor that takes into account the width of the 2DEG is significantly smaller than 1, and should cause a significant *reduction* of the SCD even at relatively high temperature and densities.

A second problem is that the RPA on which most calculations of SCD [9, 10, 12] so far have been based, neglects some important physical effects. Namely, in RPA one ignores the fact that the presence of an electron at a point in space reduces the probability of other electrons coming nearby, via the so-called exchange-correlation (xc) hole [15]. The repercussions of this effect upon the effective electron-electron interaction are quite subtle because the xc hole, while keeping the electrons apart, also reduces the collective screening of the interaction. Recently we have studied the effect of the xc hole in charge Coulomb drag and found that its introduction results in a significant *enhancement* of the charge transresistivity and qualitative changes in its temperature dependence [14]. The sheer size of these effects suggests that a quantitative description of the SCD cannot be achieved by a theory that does not include xc effects.

In this Letter we report the results of our calculations of the SCD taking into account (i) the finite transverse width of the quantum well and (ii) xc effects beyond the RPA [23], [24]. Following Ref. 14, we have used many-body local field factors [15] to take into account the modifications to the effective electron-electron interaction [16] due to the xc hole. The main outcome of our study is that the enhancement of spin drag caused by the many-body local field factors largely compensates the reduction of the effect coming from the finite well width, thereby

restoring good agreement with the experimental observations of Ref. 7.

SCD manifests itself when a current of active spin-down electrons, J_\downarrow , through the Coulomb interaction, transfers momentum to the passive spin-up electrons with zero current, $J_\uparrow = 0$. This process induces a gradient of electrochemical potential, which results in a “spin electric field” E_\uparrow . The direct measure of SCD is the spin drag resistivity, defined as

$$\rho_{\uparrow\downarrow}(T) = E_\uparrow/J_\downarrow \text{ when } J_\uparrow = 0. \quad (1)$$

We calculate the spin drag resistivity in the ballistic regime as appropriate for the clean samples used in the experiments [7]. The spin transresistivity is negative. Its absolute value at temperature T is given by [8]

$$\rho_{\uparrow\downarrow}(T) = \frac{\hbar^2}{2e^2 n_\uparrow n_\downarrow T A} \sum_{\vec{q}} q^2 \int_0^\infty \frac{d\omega}{2\pi} |W_{\uparrow\downarrow}(q, \omega)|^2 \times \frac{\text{Im}\Pi_{0\uparrow}(q, \omega)\text{Im}\Pi_{0\downarrow}(q, \omega)}{\sinh^2(\hbar\omega/2T)}, \quad (2)$$

where $\hbar\omega$ and $\hbar\vec{q}$ are, respectively, the transferred energy and in-plane momentum from the spin $\sigma = \uparrow$ to the spin $\sigma = \downarrow$, $W_{\uparrow\downarrow}(q, \omega)$ is the dynamically screened effective interaction between electrons with spins \uparrow and \downarrow , $\Pi_{0\sigma}(q, \omega)$ is the spin- σ component of the non-interacting finite temperature polarization function [15], n_\uparrow and n_\downarrow are the densities of up and down spins respectively, and A is the area of the sample. Further we will assume that the electron gas has no net spin polarization, i.e. $n_\uparrow = n_\downarrow = n/2$.

To calculate the effective interaction $W_{\uparrow\downarrow}(\omega)$ we have used an approximation scheme developed a few years ago by Vignale and Singwi (VS) [16] following the pioneering treatment of Kukkonen and Overhauser [17]. Within this scheme the effective electron-electron interaction, $\hat{W}(q, \omega)$ (a 2×2 symmetric matrix with components $W_{\uparrow\uparrow}$, $W_{\uparrow\downarrow}$, and $W_{\downarrow\downarrow}$) is approximated as

$$\hat{W}(q, \omega) = \hat{v}(q) - \hat{V}(q, \omega) \cdot \hat{\Pi}(q, \omega) \cdot \hat{V}(q, \omega) \quad (3)$$

where the full polarization function, $\hat{\Pi}(q, \omega)$, is defined in terms of the non-interacting polarization function, $\hat{\Pi}_0(q, \omega)$, and the unscreened effective Coulomb interactions, $\hat{V}(q, \omega)$ (defined below) has the form

$$\hat{\Pi}(q, \omega) = \hat{\Pi}_0(q, \omega) \cdot \left[1 + \hat{V}(q, \omega) \cdot \hat{\Pi}_0(q, \omega) \right]^{-1}. \quad (4)$$

Here all quantities are 2×2 matrices and the dot denotes a matrix product. Selecting the $\uparrow\downarrow$ component we get

$$W_{\uparrow\downarrow}(q, \omega) = V_{\uparrow\downarrow}(q, \omega)/\varepsilon(q, \omega) + v(q)G_{\uparrow\downarrow}(q, \omega)F(qd), \quad (5)$$

where the unscreened effective Coulomb interaction between spins σ and σ' is

$$V_{\sigma\sigma'}(q, \omega) = v(q) (1 - G_{\sigma\sigma'}(q, \omega)) F(qd), \quad (6)$$

where d is the width of the well. Observe how the unscreened effective interaction $V_{\sigma\sigma'}(q, \omega)$ differs from the bare Coulomb interaction $v(q) = 2\pi e^2/\kappa_0 q$ in two ways. First, the spin-resolved local field factors $G_{\sigma\sigma'}$ reduce the bare interaction by a factor of $1 - G_{\sigma\sigma'}(q, \omega)$: this is the effect of the xc hole, mentioned in the introduction. Second, the form factor $F(qd)$ takes into account the form of the wave function of the lowest transverse subband, and is defined as the square of the Fourier transform of the corresponding density profile $\rho(z)$ (for $\rho(z) = (2/d) \sin(\pi z/d)^2$ we have $F(qd) = 1 - (1/3 - 5/4\pi^2)qd$ for $qd \rightarrow 0$ and $F(qd) \rightarrow 3/(4\pi^2 qd)$ for $qd \rightarrow \infty$). Taking $d = 12$ nm and $q \simeq k_F$ we have $qd \simeq 2$ for $n = 4.3 \cdot 10^{11} \text{ cm}^{-2}$ so that $|F(qd)|^2 \simeq 0.5$.

In Eq. (5) $\varepsilon(q, \omega)$ is an effective dielectric function, which can be represented in the following manner

$$\varepsilon(q, \omega) = (1 + V_+(q, \omega)\Pi_0(q, \omega)) (1 + V_-(q, \omega)\Pi_0(q, \omega)),$$

where $V_\pm(q, \omega) \equiv (V_{\uparrow\uparrow}(q, \omega) \pm V_{\uparrow\downarrow}(q, \omega))/2$. Introducing the corresponding notation for the local field factors, $G_\pm \equiv (G_{\uparrow\uparrow} \pm G_{\uparrow\downarrow})/2$, we can write

$$V_+(q, \omega) = v(q) (1 - G_+(q, \omega)) F(qd), \quad (7)$$

$$V_-(q, \omega) = -v(q)G_-(q, \omega)F(qd) \quad (8)$$

G_+ and G_- are known as the “charge-channel” and the “spin-channel” local field factors respectively.

In our actual calculations, described below, the local field factors are derived from diffusion Monte Carlo simulations [18, 19] in the low-energy finite-momentum transfer regime, and from a combination of analytical and empirical methods [20, 21] in the small-momentum, high-energy transfer regime. These two regimes correspond to two distinct mechanism of electron-electron interaction: in the first, the interaction is mediated by electron-hole pairs, and in the second by plasmons [25].

Notice that both the dynamic and static local field factors [19, 21], which we have used in our calculations, are evaluated at $T = 0$ for an ideal 2DEG of zero width. At present there are no calculations of the local field factors that take into account the finite width of the quantum well and/or the temperature dependence. We expect, however, that the temperature dependence of the local field factors will play a relatively minor role in comparison to the temperature dependence that we explicitly include in the Fermi factors and in the non-interacting polarization functions. Similarly, the effect of the finite width of the well should be largely taken care of by our use of the form factors $F(qd)$ in Eqs. (5) and (6).

Recently Weber *et al.* [7] have demonstrated experimentally that the spin diffusion constant of electrons in an n-type GaAs quantum well is strongly reduced by the Coulomb interaction over a broad range of temperatures and electron densities. Their measurements were in good agreement with the theoretical calculations of D’Amico

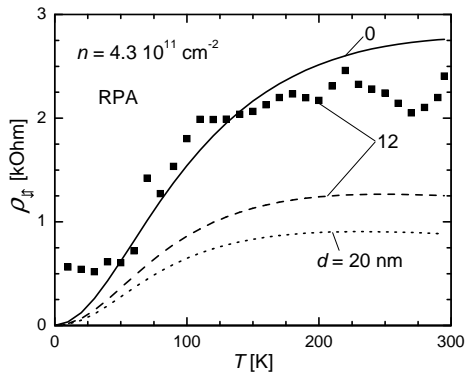


FIG. 1: Spin drag resistivity $\rho_{\uparrow\downarrow}$ as a function of temperature, calculated within RPA. The solid, dashed, and dotted curves correspond, respectively, to $\rho_{\uparrow\downarrow}$ for $d = 0, 12$, and 20 nm. The black squares represent $\rho_{\uparrow\downarrow}$, deduced from the experimental data reported in Fig. 3 of Ref. 7 for the sample with $n = 4.3 \cdot 10^{11} \text{ cm}^{-2}$ and $d = 12$ nm.

and Vignale [10], which, however, neglected both the finite width of the quantum well and the local field factor. Here we present results of calculations which include both these effects.

First we show that the effect of the finite width of the quantum well is very important. In Fig. 1 we plot the absolute value of the spin drag resistivity, calculated within RPA, vs temperature for three different values of the width, $d = 0, 12$, and 20 nm. The solid curve reproduces the RPA calculations of the spin drag resistivity for an ideal 2DEG of zero width [10]. The black squares show the spin drag resistivity, as deduced from the experimentally measured spin diffusion constant D_s according to the formula [7, 22]

$$\frac{D_c}{D_s} = \frac{\rho}{\rho + \rho_{\uparrow\downarrow}} \frac{\Pi_s}{\Pi_0}. \quad (9)$$

Here the charge diffusion constant D_c is obtained from the electric resistivity ρ via the Einstein relation, and Π_s/Π_0 is the many-body enhancement of the spin susceptibility of the electron gas, which, for the time being, following Ref. 7, we approximate as 1. Fig. 1 shows that an increase in the width of the quantum well is accompanied by a strong reduction of the spin drag resistivity: at $T = T_F$ ($T_F \simeq 178$ K) $\rho_{\uparrow\downarrow}$ for $d = 12$ nm is a factor of two smaller than at $d = 0$. The physical reason for this effect is that the SCD is dominated by large angle scattering events, where the momentum transfer q is of the order of the diameter of the Fermi sphere. At these values of q the form factor $F(qd)$ is substantially smaller than 1 – the more so the higher the density. By contrast, in the ordinary charge Coulomb drag large angle scattering events are exponentially suppressed for $q > 1/\Lambda$ where Λ is the inter-layer separation. Then, the finite width of the quantum well produces corrections of the order of $|F(d/\Lambda)|^2$, which is rather close to 1.

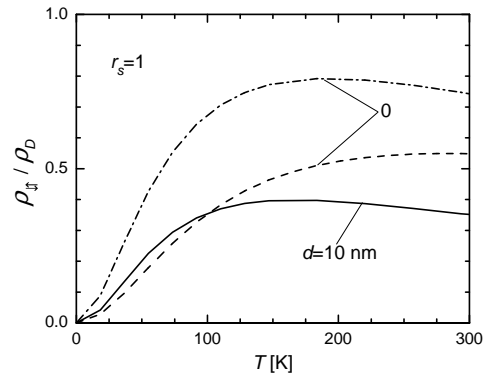


FIG. 2: The scaled spin drag resistivity $\rho_{\uparrow\downarrow}/\rho_D$ vs T for $r_s = 1$. The solid and dash-dotted curves are calculated for a 2DEG of the width $d = 0$ and 10 nm within the VS scheme. The dashed curve corresponds to $\rho_{\uparrow\downarrow}/\rho_D$ of an ideal 2DEG, calculated within the RPA. The ordinary Drude resistivity, ρ_D , is calculated for the mobility $\mu = 300 \text{ V cm}^{-1} \text{ s}^{-1}$.

Next we include the effect of exchange and correlation on the spin drag resistivity. In Fig. 2 we compare the spin drag resistivity, calculated within the VS scheme, with the corresponding result obtained within the RPA. The scaled spin drag resistivity is shown as a function of T for the dimensionless parameter $r_s = 1$ corresponding to $n = 3.5 \cdot 10^{11} \text{ cm}^{-2}$ and to the Fermi temperature $T_F = 147$ K. Notice that the inclusion of the local field factors shifts the peak of the spin drag resistivity towards lower temperatures compared to the RPA. Accordingly, the spin drag resistivity curves show quite different slopes at high temperatures depending on whether local field factors are included or not. As a result, the reduction in spin drag coming from the finite width of the well is largely compensated by the many-body enhancement of the spin drag resistivity. The situation is similar for $r_s = 2$. We conclude that the combined effect of the finite width and the xc hole on the effective interaction restores the agreement between theory and experiment.

In Fig. 3 we verify this conclusion for the strict experimental situation of Ref. 7. Here the dotted and dashed curves are calculated within RPA, respectively, for $d = 0$ and 12 nm and show the strong reduction of spin drag due to the form factor of the quantum well. The spin drag resistivity is deduced from the experimental data for the spin diffusion via Eq. (9). However, we have now taken into account the many-body enhancement of the spin susceptibility with respect to the non-interacting spin susceptibility. In a paramagnetic electron liquid the Hartree terms cancel and the enhancement is determined by the “spin-channel” many-body local field factor $G_-(q, 0)$ according to the formula $\Pi_0/\Pi_s = 1 + \lim_{q \rightarrow 0} V_-(q, 0)\Pi_0(q, 0)$ [15]. Then, it is seen that our theoretical curve for the spin drag resistivity, including both the many-body effects beyond RPA and the finite width of the quantum well, agrees very well with

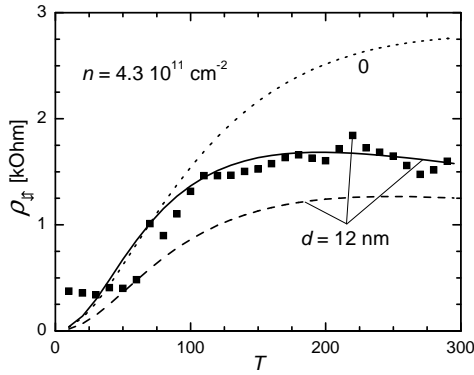


FIG. 3: Spin drag resistivity ρ_{\perp} as a function of temperature for the electron density $n = 4.3 \cdot 10^{11} \text{ cm}^{-2}$. The dashed and dotted curves correspond to ρ_{\perp} , calculated within the RPA for $d = 0$ and 12 nm , respectively. The solid curve correspond to ρ_{\perp} , calculated beyond the RPA within the VS scheme for $d = 12 \text{ nm}$. The symbols represent ρ_{\perp} , deduced from the experimental data for $d = 12 \text{ nm}$ of Ref. 7. Notice that the data points for the spin drag resistivity are lower than in Fig. 1, because we have taken into account the many-body enhancement of the spin susceptibility in converting the experimental values of D_s to ρ_{\perp} according to Eq. (9).

the experimental data of Ref. [7], while the RPA, in the presence of the form factor, does not.

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- [23] Our calculations do not include the spin-orbit interaction in the one-electron states because the measurement of Ref. [7] were done on a spin grating with wave vector along the [11] direction of GaAs, which is the direction along which spin-orbit effects are essentially non existent. We might add that even in different directions the influence of the spin-orbit coupling on the spin diffusion constant would show up as a significant effect only at wave vectors much smaller than the one studied in Ref. [7]. Furthermore, according to a recent calculation in Ref. [12], the spin Coulomb drag is enhanced by Rashba spin-orbit coupling, γ , by a small factor $1 + \gamma^2$, which for the typical value of $\gamma \sim 0.1$, is a 1% correction.
- [24] We have neglected the phonon-mediated SCD. In a single electron layer the phonon-mediated interaction between electrons cannot compete with the Coulomb interaction, due to the smallness of the electron-phonon coupling constant. The phonon-mediated interaction is effective only in bilayers with large barrier thickness, where the inter-layer Coulomb interaction becomes very small.
- [25] In the static regime the local field factors are deduced from the computed static response of 2DEG to periodic external potentials of wave vector q : they vary linearly with q both at small q and at large q but with different slopes as discussed in Section 5.6 of Ref. [15]. The local field factor in the dynamical regime is obtained from a perturbative mode-coupling theory, described in [20, 21].